$\sum_i \sum_R (t_R c_{i\uparrow}^\dagger c_{i+R\downarrow} + t_R^* c_{i+R\downarrow}^\dagger c_i)$

susy

$$H=\sum_{ij}(t_{ij}\, c_{i\uparrow}^\dagger c_{j\downarrow} + t_{ij}^*\, c_{j\downarrow}^\dagger c_{i\uparrow}) + U\sum_i (n_{i\uparrow}-\tfrac12)(n_{i\downarrow}-\tfrac12)$$

$$t_{ij}=t_R\ (R=r_i-r_j) = i\,\frac{(-1)^{R_x}}{\frac{L_x}{\pi}\sin(\frac{\pi R_x}{L_x})}\,\delta_{R_y,0} - \frac{(-1)^{R_y}}{\frac{L_y}{\pi}\sin(\frac{\pi R_y}{L_y})}\,\delta_{R_x,0}$$

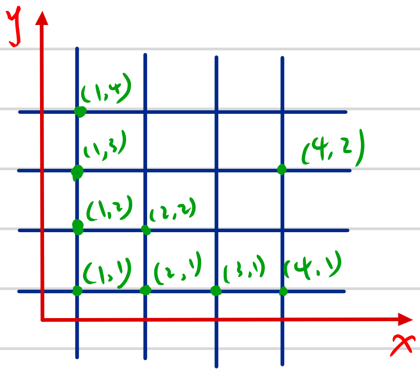
$R_x=1,\cdots,L_x-1$; $R_y=1,\cdots,L_y-1$

L_x: x方向上的格点数

L_y: y方向上的格点数

$U<0$

方形晶格



元胞=格点。

可观测量　结构因子 $S_{sc}(L)=\frac{1}{L^4}\sum_{ij}\langle\Delta_i^\dagger\Delta_j\rangle = \frac{1}{L^4}\sum_{ij}\langle c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{j\downarrow}c_{j\uparrow}\rangle$, $\Delta_i = c_{i\downarrow}c_{i\uparrow}$

相当于格点数平方 N^2

格点指标

Binder ratio $B\equiv \frac{M_4}{M_2^2}$, $M_2\equiv\frac{1}{N^2}\sum_{ij}\langle\Delta_i^\dagger\Delta_j\rangle = S_{sc}$, $M_4\equiv\frac{1}{N^4}\sum_{ijkl}\langle\Delta_i^\dagger\Delta_j^\dagger\Delta_k\Delta_l\rangle$

(RG不变量，在U_c处与L无关)

$M_2\propto L^{-(1+\eta_b)}$, η_b: boson反常维度

$G_f(L)=\frac{1}{L^2}\sum_i\langle c_i^\dagger c_{i+R_m}+c_{i+R_m}^\dagger c_i\rangle \propto L^{-(2+\eta_f)}$, R_m: $(\frac{L-1}{2},\frac{L-1}{2})$, η_f: fermion反常维度

※对于动能项　$t_R = i\frac{\pi}{L_x}\frac{1}{\sin(\frac{\pi R_x}{L_x})}(-1)^{R_x}\delta_{R_y,0} - \frac{\pi}{L_y}\frac{1}{\sin(\frac{\pi R_y}{L_y})}(-1)^{R_y}\delta_{R_x,0}$

$\delta_{R_y,0}$ 和 $\delta_{R_x,0}$ ⇒ 只可能往x轴或y轴方向上跃迁

考虑 $L\to\infty$ ($L_x,L_y\to\infty$)，对于最近邻格点.
$$t_R=\begin{cases} i\frac{\pi}{L_x}\frac{1}{\frac{\pi}{L_x}}(-1) = -i & R_x=1,\ R_y=0\\ 1 & R_x=0,\ R_y=1\end{cases}$$

对于次近邻格点.
$$t_R=\begin{cases} i\frac{\pi}{L_x}\frac{1}{\frac{2\pi}{L_x}}(-1)^2 = \frac12 i & R_x=2,\ R_y=0\\ -\frac12 & R_x=0,\ R_y=2\end{cases}$$

对于最远格点.
$$t_R=\begin{cases} i\frac{\pi}{L_x}\frac{1}{\sin\frac{\pi}{2}}(-1)^{\frac{L_x}{2}} = 0 & R_x=\frac{L_x}{2},\ R_y=0\\ 0 & R_x=0,\ R_y=\frac{L_y}{2}\end{cases}$$

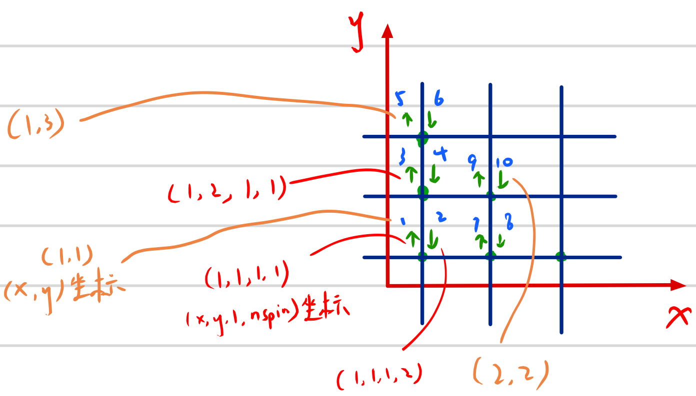
跃迁的同时伴随自旋翻转

※对于 $U<0$　HS变换如何改变　　辅助场不变

※对于方形晶格 $\begin{cases}\text{不再有元胞与格点的区分，物理量的定义要发生改变}\\ \text{SLI 设置晶格，可跃迁的格点为 } (L_x-1)+(L_y-1) \text{ 个}\end{cases}$

※测量3个可观测量 M_2, M_4, $G_f(L)$

SLI



list(格点序号, 第几个坐标变)
用于输出坐标数值

nlist(自由度序号, 第几个坐标变) 总数为ndim
用于输出坐标数值

invlist(x, y)
用于输出坐标对应格点序号

invnlist(x, y, l, nspin)
用于输出坐标对应自由度序号

L_bonds(格点序号, 0/1/2/3/4)

记为格点i
i上方的最近邻格点
i右方的最近邻格点
i下方的最近邻格点
i左方的最近邻格点

附有 period boundary condition

※ 单位矩阵 $ZKRON_{NDIM\times NDIM}$

※ 函数 Iscalar(vec1, vec2) 二阶向量点乘

NPBCX(NR) NR > nlx 则输出 NR - nlx, NR < 1 则输出 NR + nlx, 否则输出 NR

NPBCY(NR) 同上

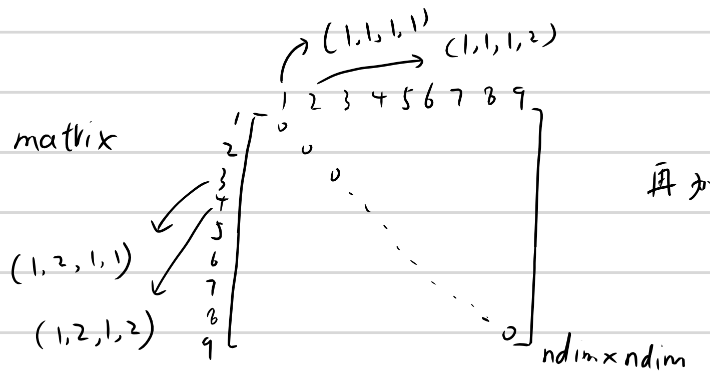
NPBC_SPIN(NR) NR > nspin 则输出 NR - nspin, NR < 1 则输出 NR + nspin, 否则输出 NR

NPBC_RX(NR) NR = nlx-1 则输出 1, 否则输出 NR

NPBC_RY(NR) 同上

SetH

hopping matrix



再加上随机微扰

$$ndim = \underbrace{\underset{\text{元胞数}}{LQ}\cdot \underset{1}{norb}}_{\text{格点数}}\cdot \underset{2}{nspin}$$ 为矩阵维数

SetHproj 略

SALPH

for Hubbard, 若 $U\geq 0$, 则 alpha_U 记为 $\alpha_U = i\sqrt{U\cdot\Delta\tau}$ (N_SUN = 1)

若 $U\leq 0$, 则 $\alpha_U = \sqrt{-U\cdot\Delta\tau}$

对于 $U>0$, α_U 是虚数, $e^{-\Delta\tau U O^2} = \sum e^{\sqrt{\Delta\tau U}\,\eta(l) O}\cdots$

对于 $U<0$, α_U 是实数, $e^{-\Delta\tau U O^2} = \sum e^{\sqrt{\Delta\tau U}\,\eta(l) O}\cdots$

$XSIGMA\_U_{up}(l) = e^{\alpha_{U\uparrow}\cdot\eta(l)}$

$XSIGMA\_U_{do}(l) = e^{\alpha_{U\downarrow}\cdot\eta(l)}$

$DELTA\_U_{up}(l=-2/-1/1/2,\ 1/2/3) = e^{\alpha_{U\uparrow}[\eta(l')-\eta(l)]}-1$

$DELTA\_U_{do}(l=-2/-1/1/2,\ 1/2/3) = e^{\alpha_{U\downarrow}[\eta(l')-\eta(l)]}-1$

$DETA\_U(l=-2/-1/1/2,\ 1/2/3) = e^{-\frac{1}{2}\alpha_U[\eta(l')-\eta(l)]}$

S proj

把 SetHproj 导出的 HLP2 记为 TMP, Diag(TMP, PROJ, wc)

$$TMP_{ndim\times ndim}|n\rangle_{ndim\times 1} = n|n\rangle_{ndim\times 1}$$

这里跟 Assaad 讲义上一样了, $ndim = x = (i, \sigma)$

各个 $|n\rangle$ 组成 PROJ 试探波函数

wc 是本征值表 (ndim 维向量)

PROJ $\begin{bmatrix} \vdots & \vdots & \vdots & \vdots \end{bmatrix}$

每一列都是一个本征向量

EN_FREE = 前 NE 个本征值相加

DEGEN = wc(NE+1) − wc(NE)

☆ L 取奇数使得动量能取到 Dirac 点.

☆ 因为要半满, 电子数要为格点数 ±1

St Hop
INconfc
MMTHR
MMTHL

MMTHLM1
MMTHRM1
ORTHO
CALC gr

略

MMUUR

输入 A

输出 $e^{\alpha_U \cdot \eta(NSIGL\_U)}\cdot A$ 即 $e^{\alpha_{U\uparrow}\cdot\eta(\pm1)}\cdot A$ 或 $e^{\alpha_{U\downarrow}\cdot\eta(\pm1)}\cdot A$ (矩阵元对应自旋↑则乘 $e^{\alpha_{U\uparrow}\cdot\eta(\pm1)}$, 反之亦然)

判断条件 U>0 要改为 U<0

MMUUL

输入 A

输出 $A\cdot e^{\alpha_U\cdot\eta(NSIGL\_U)}$ 即 $A\cdot e^{\alpha_{U\uparrow}\cdot\eta(\pm1)}$ 或 $A\cdot e^{\alpha_{U\downarrow}\cdot\eta(\pm1)}$

MMUURM1

输入 A

输出 $A / e^{\alpha_U\cdot\eta(NSIGL\_U)}$ 即 $A / e^{\alpha_{U\uparrow}\cdot\eta(\pm1)}$ 或 $A / e^{\alpha_{U\downarrow}\cdot\eta(\pm1)}$

$l_{i,\tau}$

MMUULM1

输入 A

输出 $A / e^{\alpha_U\cdot\eta(NSIGL\_U)}$ 即 $A / e^{\alpha_{U\uparrow}\cdot\eta(\pm1)}$ 或 $A / e^{\alpha_{U\downarrow}\cdot\eta(\pm1)}$

$l_{i,\tau}$

UPGRADE U

在格点 i 和虚时间 τ 下

由 iseed0 随机

$$DEL44 = DELTA\_Uup\,(\,l_{i,\tau}=\pm 1\ ,\ 1/2/3\,) = e^{\alpha_{U\uparrow}\cdot[\eta(l'_{i,\tau})-\eta(l_{i,\tau})]}-1$$

$$DEL55 = DELTA\_Udo\,(\,l_{i,\tau}=\pm 1\ ,\ 1/2/3\,) = e^{\alpha_{U\downarrow}\cdot[\eta(l'_{i,\tau})-\eta(l_{i,\tau})]}-1$$

$$VHLP1(电子) = \left\{e^{\alpha_{U\uparrow}\cdot[\eta(l'_{i,\tau})-\eta(l_{i,\tau})]}-1\right\}\cdot UR\,(自由度\ i\uparrow\ ,\ 电子)$$

$$VHLP2(电子) = \left\{e^{\alpha_{U\downarrow}\cdot[\eta(l'_{i,\tau})-\eta(l_{i,\tau})]}-1\right\}\cdot UR\,(自由度\ i\downarrow\ ,\ 电子)$$

$$UHLP1(电子) = UL\,(电子,\ 自由度\ i\uparrow)$$

$$UHLP2(电子) = UL\,(电子,\ 自由度\ i\downarrow)$$

$$G44up \mathrel{+}= \overbrace{\left\{e^{\alpha_{U\uparrow}\cdot[\eta(l'_{i,\tau})-\eta(l_{i,\tau})]}-1\right\}\cdot UR\,(自由度\ i\uparrow\ ,\ nl\,)\cdot ULRINV(nl\ ,\ 电子)}^{V1_{电子} = \Delta_\uparrow^{(i)}\, B^>_{i\uparrow,nl}\,(B^<B^>)^{-1}_{nl,电子}}\cdot UL\,(电子,\ 自由度\ i\uparrow)$$

$$= \Delta_\uparrow^{(i)}\left[B(\tau,0)\,P\cdot(P^\dagger B(2\theta,0)P)^{-1}\,P^\dagger B(2\theta,\tau)\right]_{i\uparrow,i\uparrow}$$

$$= \Delta_\uparrow^{(i)}\left[B^>(B^<B^>)^{-1}B^<\right]_{i\uparrow,i\uparrow}$$

$$G54up = \Delta_\downarrow^{(i)}\left[B^>(B^<B^>)^{-1}B^<\right]_{i\downarrow,i\uparrow}$$

$$G45up = \Delta_\uparrow^{(i)}\left[B^>(B^<B^>)^{-1}B^<\right]_{i\uparrow,i\downarrow}$$

$$G55up = \Delta_\downarrow^{(i)}\left[B^>(B^<B^>)^{-1}B^<\right]_{i\downarrow,i\downarrow}$$

$$RATIOup = (1+G44up)\cdot(1+G55up) - G45up\cdot G54up$$

$$RATIOtot = \frac{\gamma(l')}{\gamma(l)}\cdot RATIOup\cdot e^{-\frac{1}{2}\alpha_U[\eta(l'_{i,\tau})-\eta(l_{i,\tau})]}$$

$$RATIO\_abs = |RATIOtot|$$

对负数取绝对值会怎样

HS 变换准备与结果

要把 $U\sum_i(n_{i\uparrow}-\frac{1}{2})(n_{i\downarrow}-\frac{1}{2})$ 变为 $H_1=-U\sum_i(O^{(i)})^2$ 的形式：

$$\begin{cases}(n_{i\uparrow}+n_{i\downarrow}-1)^2 = (n_{i\uparrow}+n_{i\downarrow})^2-2(n_{i\uparrow}+n_{i\downarrow})+1 = n_{i\uparrow}^2+n_{i\downarrow}^2+2n_{i\uparrow}n_{i\downarrow}-2(n_{i\uparrow}+n_{i\downarrow})+1\\ n_{i\uparrow}^2 = c_{i\uparrow}^\dagger c_{i\uparrow}c_{i\uparrow}^\dagger c_{i\uparrow} = c_{i\uparrow}^\dagger(1-c_{i\uparrow}^\dagger c_{i\uparrow})c_{i\uparrow} = c_{i\uparrow}^\dagger c_{i\uparrow} = n_{i\uparrow}\end{cases}$$

$$\Rightarrow (n_{i\uparrow}+n_{i\downarrow}-1)^2 = 2n_{i\uparrow}n_{i\downarrow}-(n_{i\uparrow}+n_{i\downarrow})+1 = 2(n_{i\uparrow}-\tfrac{1}{2})(n_{i\downarrow}-\tfrac{1}{2})+\tfrac{1}{2}$$

$$e^{\delta\tau U(n_{i\uparrow}-\frac{1}{2})(n_{i\downarrow}-\frac{1}{2})} = e^{\frac{1}{2}\delta\tau U(n_{i\uparrow}+n_{i\downarrow}-1)^2}\,e^{-\frac{1}{4}\delta\tau U} = e^{-\frac{1}{4}\delta\tau U}\sum_l\gamma(l)\,e^{\frac{1}{2}\sqrt{\delta\tau U}\,\eta(l)\,(n_{i\uparrow}+n_{i\downarrow}-1)}$$

对于 sampling，只需考虑 $\gamma(l)\,e^{\frac{1}{2}\sqrt{\delta\tau U}\,\eta(l)\,(n_{i\uparrow}+n_{i\downarrow})}\,e^{-\frac{1}{2}\sqrt{\delta\tau U}\,\eta(l)}$

The MC Sampling

$$Pr_i = \frac{C_i\ \det(P^\dagger B_i(2\theta,0)P)}{\sum_i C_i\det(P^\dagger B_i(2\theta,0)P)}$$

$$R = \frac{Pr_{i'}}{Pr_i} = \frac{C_{i'}}{C_i}\,\frac{\det(P^\dagger B_{i'}(2\theta,0)P)}{\det(P^\dagger B_i(2\theta,0)P)} = \frac{\gamma(l')}{\gamma(l)}\det\left[I+\Delta^{(i)}B_i^>(B_i^<B_i^>)^{-1}B_i^<\right]$$

然后把 $\Delta^{(i)}B_i^>(B_i^<B_i^>)^{-1}B_i^<$ 写成 2x2 矩阵

$$\begin{pmatrix}\Delta_\uparrow^{(i)}[B^>(B^<B^>)^{-1}B^<]_{i\uparrow,i\uparrow} & \Delta_\uparrow^{(i)}[B^>(B^<B^>)^{-1}B^<]_{i\uparrow,i\downarrow}\\ \Delta_\downarrow^{(i)}[B^>(B^<B^>)^{-1}B^<]_{i\downarrow,i\uparrow} & \Delta_\downarrow^{(i)}[B^>(B^<B^>)^{-1}B^<]_{i\downarrow,i\downarrow}\end{pmatrix} = \begin{pmatrix}G44up & G45up\\ G54up & G55up\end{pmatrix}$$

行列式是 2x2 的是因为一次只改变一个位置的辅助场，Δ 只有这个位置 ↑ 和 ↓ 的那两个元素非零。

upgrade the inverse

若 RATIO_abs > RANF(ISEED0) 则 ACCM +=1

weight = 模(RATIO tot)

RATIOtot 的模用来计算蒙卡更新概率

U1(电子) += ULRINV(电子, nl) · UL(nl, 自由度 i↑)

U2(电子) += ULRINV(电子, nl) · UL(nl, 自由度 i↓)

$$Z1 = \frac{1}{1+G55up} \qquad Z2 = \frac{G54up}{1+G55up} \qquad Z3 = \frac{G45up}{1+G55up} \qquad Z4 = \frac{1+G55up}{RATIO\ up}$$

$$UHLP1(电子) = U2(电子)$$

$$UHLP2(电子) = \left[U1(电子) - U2(电子) \cdot \frac{G54up}{1+G55up}\right] \cdot \frac{1+G55up}{RATIO\ up}$$

$$VHLP1(电子) = V2(电子) \cdot \frac{1}{1+G55up}$$

$$VHLP2(电子) = V1(电子) - V2(电子) \cdot \frac{G45up}{1+G55up}$$

$$ULRINV = ULRINV - UHLP1 \cdot VHLP1 - UHLP2 \cdot VHLP2$$

For the PQMC, the upgrading of the Green function is equivalent to the upgrading of $(B^{\langle}_{\vec{s}'} B^{\rangle}_{\vec{s}'})^{-1}$, which is achieved with the Sherman-Morrison formula:

$$(B^{\langle}_{\vec{s}'} B^{\rangle}_{\vec{s}'})^{-1} = \left(B^{\langle}_{\vec{s}}(1+\Delta^{(\vec{i})})B^{\rangle}_{\vec{s}}\right)^{-1} = \left(B^{\langle}_{\vec{s}} B^{\rangle}_{\vec{s}} + \sum_q \vec{u}^{(q)} \otimes \vec{v}^{(q)}\right)^{-1} \qquad (66)$$

with $(\vec{u}^{(q)})_x = (B^{\langle}_{\vec{s}})_{x,x_q} \Delta^{(\vec{i})}_{x_q}$ and $(\vec{v}^{(q)})_x = (B^{\rangle}_{\vec{s}})_{x_q,x}$. Here x runs from $1 \cdots N_p$ where N_p corresponds to the number of particles contained in the trial wave function.

upgrade UR

UR(i↑, 电子) = (1+DEL44) · UR(i↑, 电子)

UR(i↓, 电子) = (1+DEL55) · UR(i↓, 电子)

flip

由 iseed0 随机

NSIGL_U(i, τ) = NFLIPL($l_{i,\tau}$, 1/2/3)

Cummulants , Wicks Theorem and Observables

$$G(\eta_4) = 1 - B^{>}(B^{<} e^{\eta_4 \Lambda^{(4)}} B^{>})^{-1} B^{<} e^{\eta_4 \Lambda^{(4)}}$$

$$\frac{\partial}{\partial \eta_4} G(\eta_4)\Big|_{\eta_4=0} = -\bar{G}\Lambda^{(4)} G$$

$$\frac{\partial}{\partial \eta_4} \overline{G(\eta_4)}\Big|_{\eta_4=0} = \bar{G}\Lambda^{(4)} G$$

$$\begin{aligned}
\langle\langle O_4 O_3 O_2 O_1 \rangle\rangle &= \frac{\partial}{\partial \eta_4} \mathrm{Tr}\Big(\overline{G(\eta_4)}\Lambda^{(3)} G(\eta_4)\Lambda^{(2)} G(\eta_4)\Lambda^{(1)} \\
&\qquad - \overline{G(\eta_4)}\Lambda^{(3)} G(\eta_4)\Lambda^{(1)} \overline{G(\eta_4)}\Lambda^{(2)} \Big)\Big|_{\eta_4=0} \\
&= \mathrm{Tr}\Big(\bar{G}\Lambda^{(4)} G\Lambda^{(3)} G\Lambda^{(2)} G\Lambda^{(1)} \\
&\qquad - \bar{G}\Lambda^{(3)} \bar{G}\Lambda^{(4)} G\Lambda^{(2)} G\Lambda^{(1)} \\
&\qquad - \bar{G}\Lambda^{(3)} G\Lambda^{(2)} \bar{G}\Lambda^{(4)} G\Lambda^{(1)} \\
&\qquad - \bar{G}\Lambda^{(4)} G\Lambda^{(3)} G\Lambda^{(1)} \bar{G}\Lambda^{(2)} \\
&\qquad + \bar{G}\Lambda^{(3)} \bar{G}\Lambda^{(4)} G\Lambda^{(1)} \bar{G}\Lambda^{(2)} \\
&\qquad - \bar{G}\Lambda^{(3)} G\Lambda^{(1)} \bar{G}\Lambda^{(4)} G\Lambda^{(2)} \Big) \\
&= \bar{G}_{y_1x_4} G_{y_4x_3} G_{y_3x_2} G_{y_2x_1} - \bar{G}_{y_1x_3} \bar{G}_{y_3x_4} G_{y_4x_2} G_{y_2x_1} \\
&\quad - \bar{G}_{y_1x_3} G_{y_3x_2} \bar{G}_{y_2x_4} G_{y_4x_1} - \bar{G}_{y_2x_4} G_{y_4x_3} G_{y_3x_1} \bar{G}_{y_1x_2} \\
&\quad + \bar{G}_{y_2x_3} \bar{G}_{y_3x_4} G_{y_4x_1} \bar{G}_{y_1x_2} - \bar{G}_{y_2x_3} G_{y_3x_1} \bar{G}_{y_1x_4} G_{y_4x_2} \\
&= \langle c^\dagger_{x_4} c_{y_1}\rangle \langle c_{y_4} c^\dagger_{x_3}\rangle \langle c_{y_3} c^\dagger_{x_2}\rangle \langle c_{y_2} c^\dagger_{x_1}\rangle - \langle c^\dagger_{x_3} c_{y_1}\rangle \langle c^\dagger_{x_4} c_{y_3}\rangle \langle c_{y_4} c^\dagger_{x_2}\rangle \langle c_{y_2} c^\dagger_{x_1}\rangle \\
&\quad - \langle c^\dagger_{x_3} c_{y_1}\rangle \langle c_{y_3} c^\dagger_{x_2}\rangle \langle c^\dagger_{x_4} c_{y_2}\rangle \langle c_{y_4} c^\dagger_{x_1}\rangle - \langle c^\dagger_{x_4} c_{y_2}\rangle \langle c_{y_4} c^\dagger_{x_3}\rangle \langle c_{y_3} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_2} c_{y_1}\rangle \\
&\quad + \langle c^\dagger_{x_3} c_{y_2}\rangle \langle c^\dagger_{x_4} c_{y_3}\rangle \langle c_{y_4} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_2} c_{y_1}\rangle - \langle c^\dagger_{x_3} c_{y_2}\rangle \langle c_{y_3} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_4} c_{y_1}\rangle \langle c_{y_4} c^\dagger_{x_2}\rangle
\end{aligned}$$

$$\begin{aligned}
\langle O_4 O_3 O_2 O_1 \rangle &= \langle\langle O_4 O_3 O_2 O_1 \rangle\rangle + \\
&\quad \langle\langle O_3 O_2 O_1 \rangle\rangle \langle O_4 \rangle + \langle\langle O_4 O_2 O_1 \rangle\rangle \langle O_3 \rangle + \langle\langle O_4 O_3 O_1 \rangle\rangle \langle O_2 \rangle + \langle\langle O_4 O_3 O_2 \rangle\rangle \langle O_1 \rangle + \\
&\quad \langle\langle O_4 O_3 \rangle\rangle \langle\langle O_2 O_1 \rangle\rangle + \langle\langle O_4 O_2 \rangle\rangle \langle\langle O_3 O_1 \rangle\rangle + \langle\langle O_4 O_1 \rangle\rangle \langle\langle O_3 O_2 \rangle\rangle + \\
&\quad \langle O_4 \rangle \langle O_3 \rangle \langle O_2 \rangle \langle O_1 \rangle
\end{aligned}$$

Assaad讲义上(42)式有问题，待解决

$$\begin{aligned}
&= \langle c^\dagger_{x_4} c_{y_1}\rangle \langle c_{y_4} c^\dagger_{x_3}\rangle \langle c_{y_3} c^\dagger_{x_2}\rangle \langle c_{y_2} c^\dagger_{x_1}\rangle - \langle c^\dagger_{x_3} c_{y_1}\rangle \langle c^\dagger_{x_4} c_{y_3}\rangle \langle c_{y_4} c^\dagger_{x_2}\rangle \langle c_{y_2} c^\dagger_{x_1}\rangle \\
&\quad - \langle c^\dagger_{x_3} c_{y_1}\rangle \langle c_{y_3} c^\dagger_{x_2}\rangle \langle c^\dagger_{x_4} c_{y_2}\rangle \langle c_{y_4} c^\dagger_{x_1}\rangle - \langle c^\dagger_{x_4} c_{y_2}\rangle \langle c_{y_4} c^\dagger_{x_3}\rangle \langle c_{y_3} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_2} c_{y_1}\rangle \\
&\quad + \langle c^\dagger_{x_3} c_{y_2}\rangle \langle c^\dagger_{x_4} c_{y_3}\rangle \langle c_{y_4} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_2} c_{y_1}\rangle - \langle c^\dagger_{x_3} c_{y_2}\rangle \langle c_{y_3} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_4} c_{y_1}\rangle \langle c_{y_4} c^\dagger_{x_2}\rangle \\
&\quad + \langle c^\dagger_{x_3} c_{y_1}\rangle \langle c_{y_3} c^\dagger_{x_2}\rangle \langle c_{y_2} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_4} c_{y_4}\rangle - \langle c^\dagger_{x_3} c_{y_2}\rangle \langle c_{y_3} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_2} c_{y_1}\rangle \langle c^\dagger_{x_4} c_{y_4}\rangle \\
&\quad + \langle c^\dagger_{x_4} c_{y_1}\rangle \langle c_{y_4} c^\dagger_{x_2}\rangle \langle c_{y_2} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_3} c_{y_3}\rangle - \langle c^\dagger_{x_4} c_{y_2}\rangle \langle c_{y_4} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_2} c_{y_1}\rangle \langle c^\dagger_{x_3} c_{y_3}\rangle \\
&\quad + \langle c^\dagger_{x_4} c_{y_1}\rangle \langle c_{y_4} c^\dagger_{x_3}\rangle \langle c_{y_3} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_2} c_{y_2}\rangle - \langle c^\dagger_{x_4} c_{y_3}\rangle \langle c_{y_4} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_3} c_{y_1}\rangle \langle c^\dagger_{x_2} c_{y_2}\rangle \\
&\quad + \langle c^\dagger_{x_4} c_{y_2}\rangle \langle c_{y_4} c^\dagger_{x_3}\rangle \langle c_{y_3} c^\dagger_{x_2}\rangle \langle c^\dagger_{x_1} c_{y_1}\rangle - \langle c^\dagger_{x_4} c_{y_3}\rangle \langle c_{y_4} c^\dagger_{x_2}\rangle \langle c^\dagger_{x_3} c_{y_2}\rangle \langle c^\dagger_{x_1} c_{y_1}\rangle \\
&\quad + \langle c^\dagger_{x_4} c_{y_3}\rangle \langle c_{y_4} c^\dagger_{x_3}\rangle \langle c^\dagger_{x_2} c_{y_1}\rangle \langle c_{y_2} c^\dagger_{x_1}\rangle + \langle c^\dagger_{x_4} c_{y_2}\rangle \langle c_{y_4} c^\dagger_{x_2}\rangle \langle c^\dagger_{x_3} c_{y_1}\rangle \langle c_{y_3} c^\dagger_{x_1}\rangle \\
&\quad + \langle c^\dagger_{x_4} c_{y_1}\rangle \langle c_{y_4} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_3} c_{y_2}\rangle \langle c_{y_3} c^\dagger_{x_2}\rangle \\
&\quad + \langle c^\dagger_{x_4} c_{y_4}\rangle \langle c^\dagger_{x_3} c_{y_3}\rangle \langle c^\dagger_{x_2} c_{y_2}\rangle \langle c^\dagger_{x_1} c_{y_1}\rangle \\
&\quad + \langle c^\dagger_{x_4} c_{y_4}\rangle \langle c^\dagger_{x_3} c_{y_3}\rangle \langle c^\dagger_{x_2} c_{y_1}\rangle \langle c_{y_2} c^\dagger_{x_1}\rangle + \langle c^\dagger_{x_4} c_{y_4}\rangle \langle c^\dagger_{x_3} c_{y_1}\rangle \langle c_{y_3} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_2} c_{y_2}\rangle \\
&\quad + \langle c^\dagger_{x_4} c_{y_4}\rangle \langle c^\dagger_{x_3} c_{y_2}\rangle \langle c_{y_3} c^\dagger_{x_2}\rangle \langle c^\dagger_{x_1} c_{y_1}\rangle + \langle c^\dagger_{x_3} c_{y_3}\rangle \langle c^\dagger_{x_2} c_{y_2}\rangle \langle c^\dagger_{x_4} c_{y_1}\rangle \langle c_{y_4} c^\dagger_{x_1}\rangle \\
&\quad + \langle c^\dagger_{x_3} c_{y_3}\rangle \langle c^\dagger_{x_1} c_{y_1}\rangle \langle c^\dagger_{x_4} c_{y_2}\rangle \langle c_{y_4} c^\dagger_{x_2}\rangle + \langle c^\dagger_{x_2} c_{y_2}\rangle \langle c^\dagger_{x_1} c_{y_1}\rangle \langle c^\dagger_{x_4} c_{y_3}\rangle \langle c_{y_4} c^\dagger_{x_3}\rangle
\end{aligned}$$

缩并线法得到24项，
但不知道绿色这些从哪来的

$$\begin{aligned}
\langle O_3 O_2 O_1 \rangle &= \langle c^\dagger_{x_3} c_{y_3}\, c^\dagger_{x_2} c_{y_2}\, c^\dagger_{x_1} c_{y_1} \rangle \\
&= \langle c^\dagger_{x_3} c_{y_1}\rangle \langle c_{y_3} c^\dagger_{x_2}\rangle \langle c_{y_2} c^\dagger_{x_1}\rangle - \langle c^\dagger_{x_3} c_{y_2}\rangle \langle c_{y_3} c^\dagger_{x_1}\rangle \langle c^\dagger_{x_2} c_{y_1}\rangle \\
&\quad + \langle c^\dagger_{x_3} c_{y_3}\rangle \langle c^\dagger_{x_2} c_{y_1}\rangle \langle c_{y_2} c^\dagger_{x_1}\rangle + \langle c^\dagger_{x_2} c_{y_2}\rangle \langle c^\dagger_{x_3} c_{y_1}\rangle \langle c_{y_3} c^\dagger_{x_1}\rangle \\
&\quad + \langle c^\dagger_{x_1} c_{y_1}\rangle \langle c^\dagger_{x_3} c_{y_2}\rangle \langle c_{y_3} c^\dagger_{x_2}\rangle + \langle c^\dagger_{x_1} c_{y_1}\rangle \langle c^\dagger_{x_2} c_{y_2}\rangle \langle c^\dagger_{x_3} c_{y_3}\rangle
\end{aligned}$$

$$\langle c^\dagger_{x_2} c_{y_2} c^\dagger_{x_1} c_{y_1}\rangle_{\vec s} = \langle c^\dagger_{x_2} c_{y_1}\rangle_{\vec s}\langle c_{y_2} c^\dagger_{x_1}\rangle_{\vec s} + \langle c^\dagger_{x_2} c_{y_2}\rangle_{\vec s}\langle c^\dagger_{x_1} c_{y_1}\rangle_{\vec s}.$$

$C_x C_y^\dagger + C_y^\dagger C_x = \delta_{x,y}$

$$\sum_{ij}\langle c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c_{j\downarrow} c_{j\uparrow}\rangle$$
$$= \sum_{ij}\langle c^\dagger_{i\uparrow} c_{j\uparrow}\delta_{i\downarrow j\downarrow} - c^\dagger_{i\uparrow} c_{j\downarrow} c^\dagger_{i\downarrow} c_{j\uparrow}\rangle \qquad \left(c^\dagger_{i\downarrow} c_{j\downarrow} = \delta_{i\downarrow j\downarrow} - c_{j\downarrow} c^\dagger_{i\downarrow}\right)$$
$$= \sum_{ij}\left(\langle c^\dagger_{i\uparrow} c_{i\uparrow}\rangle - \langle c^\dagger_{i\uparrow} c_{j\uparrow}\rangle\langle c_{j\downarrow} c^\dagger_{i\downarrow}\rangle - \langle c^\dagger_{i\uparrow} c_{j\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{j\uparrow}\rangle\right)$$
$$= \sum_{ij}\left[\mathrm{GRupc}(i\uparrow,i\uparrow) - \mathrm{GRupc}(j\uparrow,i\uparrow)\,\mathrm{GRup}(j\downarrow,i\downarrow) - \mathrm{GRupc}(j\downarrow,i\uparrow)\cdot\mathrm{GRupc}(j\uparrow,i\downarrow)\right]$$

$$\sum_{ijkl}\langle c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c^\dagger_{j\uparrow} c^\dagger_{j\downarrow} c_{k\downarrow} c_{k\uparrow} c_{l\downarrow} c_{l\uparrow}\rangle$$
$$= \sum_{ijkl}\langle c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c^\dagger_{i\uparrow} c_{k\uparrow} c_{l\downarrow} c_{l\uparrow}\delta_{jk} - c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c_{k\downarrow} c^\dagger_{j\uparrow} c_{k\uparrow} c_{l\uparrow}\delta_{jl} + c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c_{k\downarrow} c_{l\downarrow} c^\dagger_{j\downarrow} c_{l\uparrow}\delta_{jk} - c^\dagger_{i\uparrow} c_{k\uparrow} c^\dagger_{j\uparrow} c_{l\downarrow} c^\dagger_{j\downarrow} c_{l\uparrow}\delta_{ik}$$
$$+ c^\dagger_{i\uparrow} c_{k\downarrow} c^\dagger_{i\downarrow} c_{k\uparrow} c^\dagger_{j\uparrow} c_{l\downarrow} c^\dagger_{j\downarrow} c_{l\uparrow}\rangle$$
$$= \sum_{ijkl}\langle \cancel{c^\dagger_{i\uparrow} c_{i\uparrow}} - c^\dagger_{i\uparrow}\cancel{c_{l\downarrow} c^\dagger_{i\downarrow}} c_{l\uparrow} - c^\dagger_{i\uparrow} c_{j\uparrow} c^\dagger_{i\downarrow} c_{l\downarrow} c^\dagger_{i\uparrow} c_{l\uparrow} - \cancel{c^\dagger_{i\uparrow} c_{i\uparrow}} + c^\dagger_{i\uparrow}\cancel{c_{j\downarrow} c^\dagger_{i\downarrow}} c_{j\uparrow} + c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{j\uparrow} c_{j\uparrow}$$
$$+ c^\dagger_{i\uparrow} c_{k\downarrow} c^\dagger_{i\downarrow} c_{k\uparrow} c^\dagger_{j\uparrow} c_{j\uparrow} + c^\dagger_{i\uparrow} c_{l\downarrow} c^\dagger_{i\downarrow} c_{l\uparrow} - c^\dagger_{i\uparrow} c_{j\downarrow} c^\dagger_{i\downarrow} c_{l\downarrow} c^\dagger_{j\downarrow} c_{l\uparrow} - c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{j\uparrow} c_{l\downarrow} c^\dagger_{j\downarrow} c_{l\uparrow} + c^\dagger_{i\uparrow} c_{k\downarrow} c^\dagger_{i\downarrow} c_{k\uparrow} c^\dagger_{j\uparrow} c_{l\downarrow} c^\dagger_{j\downarrow} c_{l\uparrow}\rangle$$
$$= \sum_{ijkl}\langle c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{j\uparrow} c_{j\uparrow}\rangle + \langle c^\dagger_{i\uparrow} c_{j\downarrow} c^\dagger_{i\downarrow} c_{j\uparrow}\rangle - \langle c^\dagger_{i\uparrow} c_{j\uparrow} c^\dagger_{i\downarrow} c_{k\downarrow} c^\dagger_{j\uparrow} c_{k\uparrow}\rangle + \langle c^\dagger_{i\uparrow} c_{k\downarrow} c^\dagger_{i\downarrow} c_{k\uparrow} c^\dagger_{j\uparrow} c_{j\uparrow}\rangle$$
$$- \langle c^\dagger_{i\uparrow} c_{j\downarrow} c^\dagger_{i\downarrow} c_{k\downarrow} c^\dagger_{j\downarrow} c_{k\uparrow}\rangle - \langle c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{j\uparrow} c_{k\downarrow} c^\dagger_{j\downarrow} c_{k\uparrow}\rangle + \langle c^\dagger_{i\uparrow} c_{k\downarrow} c^\dagger_{i\downarrow} c_{k\uparrow} c^\dagger_{j\uparrow} c_{l\downarrow} c^\dagger_{j\downarrow} c_{l\uparrow}\rangle$$

$$= \sum_{ijkl} \langle c^\dagger_{i\uparrow} c_{j\uparrow}\rangle\langle c_{i\uparrow} c^\dagger_{j\uparrow}\rangle + \langle c^\dagger_{i\uparrow} c_{i\uparrow}\rangle\langle c^\dagger_{j\uparrow} c_{j\uparrow}\rangle + \langle c^\dagger_{i\uparrow} c_{j\uparrow}\rangle\langle c_{j\downarrow} c^\dagger_{i\downarrow}\rangle + \langle c^\dagger_{i\uparrow} c_{j\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{j\uparrow}\rangle$$
$$- \langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{j\uparrow} c^\dagger_{i\downarrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\uparrow}\rangle + \langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c_{j\uparrow} c^\dagger_{j\uparrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle - \langle c^\dagger_{i\uparrow} c_{j\uparrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\uparrow}\rangle$$
$$- \langle c^\dagger_{i\downarrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{j\uparrow} c^\dagger_{j\uparrow}\rangle - \langle c^\dagger_{j\uparrow} c_{k\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c_{j\uparrow} c^\dagger_{i\downarrow}\rangle - \langle c^\dagger_{j\uparrow} c_{k\uparrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\uparrow} c_{j\uparrow}\rangle$$
$$+ \langle c^\dagger_{i\uparrow} c_{j\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{i\downarrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\uparrow}\rangle - \langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\uparrow}\rangle\langle c^\dagger_{i\downarrow} c_{j\uparrow}\rangle + \langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{j\uparrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\uparrow}\rangle$$
$$+ \langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{j\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\uparrow}\rangle + \langle c^\dagger_{j\uparrow} c_{j\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{i\downarrow}\rangle + \langle c^\dagger_{j\uparrow} c_{j\uparrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle$$
$$- \langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{j\downarrow} c^\dagger_{i\downarrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\downarrow}\rangle + \langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c_{j\downarrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle - \langle c^\dagger_{i\uparrow} c_{j\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\downarrow}\rangle$$
$$- \langle c^\dagger_{i\downarrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{j\downarrow} c^\dagger_{j\downarrow}\rangle - \langle c^\dagger_{j\downarrow} c_{k\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c_{j\downarrow} c^\dagger_{i\downarrow}\rangle - \langle c^\dagger_{j\downarrow} c_{k\uparrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\uparrow} c_{j\downarrow}\rangle$$
$$- \langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{i\uparrow} c^\dagger_{j\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\downarrow}\rangle + \langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c_{i\uparrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{j\uparrow} c_{k\uparrow}\rangle - \langle c^\dagger_{i\uparrow} c_{i\uparrow}\rangle\langle c^\dagger_{j\uparrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\downarrow}\rangle$$
$$- \langle c^\dagger_{j\uparrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{i\uparrow} c^\dagger_{j\downarrow}\rangle - \langle c^\dagger_{j\downarrow} c_{k\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c_{i\uparrow} c^\dagger_{j\uparrow}\rangle - \langle c^\dagger_{j\downarrow} c_{k\uparrow}\rangle\langle c^\dagger_{j\uparrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\uparrow} c_{i\uparrow}\rangle$$
$$+ \langle c^\dagger_{i\uparrow} c_{l\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{i\downarrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\uparrow}\rangle\langle c_{l\downarrow} c^\dagger_{j\downarrow}\rangle - \langle c^\dagger_{i\downarrow} c_{l\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\uparrow}\rangle\langle c_{l\downarrow} c^\dagger_{j\downarrow}\rangle$$
$$- \langle c^\dagger_{i\downarrow} c_{l\uparrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{l\downarrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\downarrow}\rangle - \langle c^\dagger_{i\uparrow} c_{l\downarrow}\rangle\langle c_{k\downarrow} c^\dagger_{i\downarrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\uparrow}\rangle$$
$$+ \langle c^\dagger_{i\downarrow} c_{l\downarrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\uparrow}\rangle - \langle c^\dagger_{i\downarrow} c_{l\downarrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{i\uparrow} c_{l\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\uparrow}\rangle$$
$$+ \langle c^\dagger_{i\downarrow} c_{l\uparrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\uparrow}\rangle\langle c_{l\downarrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle - \langle c^\dagger_{i\downarrow} c_{l\downarrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle$$
$$+ \langle c^\dagger_{i\uparrow} c_{l\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\uparrow}\rangle\langle c_{l\downarrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle - \langle c^\dagger_{i\uparrow} c_{l\downarrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\uparrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle$$
$$+ \langle c^\dagger_{i\uparrow} c_{l\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{i\downarrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\downarrow}\rangle - \langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{l\uparrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\downarrow}\rangle$$
$$+ \langle c^\dagger_{i\uparrow} c_{l\downarrow}\rangle\langle c_{k\downarrow} c^\dagger_{i\downarrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\uparrow}\rangle\langle c^\dagger_{j\downarrow} c_{l\uparrow}\rangle - \langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\uparrow}\rangle\langle c^\dagger_{i\downarrow} c_{l\downarrow}\rangle\langle c^\dagger_{j\downarrow} c_{l\uparrow}\rangle$$
$$+ \langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{i\downarrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\uparrow}\rangle\langle c_{l\downarrow} c^\dagger_{j\downarrow}\rangle + \langle c^\dagger_{i\uparrow} c_{l\downarrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\uparrow}\rangle\langle c^\dagger_{i\downarrow} c_{l\uparrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\downarrow}\rangle$$
$$+ \langle c^\dagger_{i\uparrow} c_{l\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{l\downarrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\uparrow}\rangle$$
$$+ \langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\downarrow}\rangle\langle c^\dagger_{j\downarrow} c_{l\uparrow}\rangle$$
$$+ \langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\uparrow}\rangle\langle c_{l\downarrow} c^\dagger_{j\downarrow}\rangle + \langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{l\uparrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\downarrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\downarrow}\rangle$$
$$+ \langle c^\dagger_{i\uparrow} c_{k\downarrow}\rangle\langle c^\dagger_{i\downarrow} c_{l\downarrow}\rangle\langle c_{k\uparrow} c^\dagger_{j\uparrow}\rangle\langle c^\dagger_{j\downarrow} c_{l\uparrow}\rangle + \langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle\langle c^\dagger_{j\uparrow} c_{l\downarrow}\rangle\langle c^\dagger_{i\uparrow} c_{l\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\downarrow}\rangle$$
$$+ \langle c^\dagger_{i\downarrow} c_{k\uparrow}\rangle\langle c^\dagger_{j\downarrow} c_{l\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{l\downarrow}\rangle\langle c_{k\downarrow} c^\dagger_{j\uparrow}\rangle + \langle c^\dagger_{j\uparrow} c_{l\downarrow}\rangle\langle c^\dagger_{j\downarrow} c_{l\uparrow}\rangle\langle c^\dagger_{i\uparrow} c_{k\uparrow}\rangle\langle c_{k\downarrow} c^\dagger_{i\downarrow}\rangle$$

$$= \sum_{ijkl} \mathrm{GRupc}(j\uparrow,i\uparrow)\,\mathrm{GRup}(i\uparrow,j\uparrow) + \mathrm{GRupc}(i\uparrow,i\uparrow)\,\mathrm{GRupc}(j\uparrow,j\uparrow) + \mathrm{GRupc}(j\uparrow,i\uparrow)\,\mathrm{GRup}(j\downarrow,i\downarrow) + \mathrm{GRupc}(j\downarrow,i\uparrow)\,\mathrm{GRupc}(j\uparrow,i\downarrow)$$
$$- \mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(j\uparrow,i\downarrow)\,\mathrm{GRup}(k\downarrow,j\uparrow) + \mathrm{GRupc}(k\downarrow,i\uparrow)\,\mathrm{GRup}(j\uparrow,j\uparrow)\,\mathrm{GRupc}(k\uparrow,i\downarrow) - \mathrm{GRupc}(j\uparrow,i\uparrow)\,\mathrm{GRupc}(k\uparrow,i\downarrow)\,\mathrm{GRup}(k\downarrow,j\uparrow)$$
$$- \mathrm{GRupc}(k\downarrow,i\downarrow)\,\mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(j\uparrow,j\uparrow) - \mathrm{GRupc}(k\uparrow,j\uparrow)\,\mathrm{GRupc}(k\downarrow,i\uparrow)\,\mathrm{GRup}(j\uparrow,i\downarrow) - \mathrm{GRupc}(k\uparrow,j\uparrow)\,\mathrm{GRupc}(k\downarrow,i\downarrow)\,\mathrm{GRupc}(j\uparrow,i\uparrow)$$
$$+ \mathrm{GRupc}(j\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\uparrow) - \mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\uparrow)\,\mathrm{GRupc}(j\uparrow,i\downarrow) + \mathrm{GRupc}(k\downarrow,i\uparrow)\,\mathrm{GRupc}(j\uparrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\downarrow)$$
$$+ \mathrm{GRupc}(k\uparrow,i\downarrow)\,\mathrm{GRupc}(j\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\uparrow) + \mathrm{GRupc}(j\uparrow,j\uparrow)\,\mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,i\downarrow) + \mathrm{GRupc}(j\uparrow,j\uparrow)\,\mathrm{GRupc}(k\uparrow,i\downarrow)\,\mathrm{GRupc}(k\downarrow,i\uparrow)$$

j↑全部变j↓

$$-\mathrm{GRupc}(\ ,\ )\,\mathrm{GRup}(\ ,\ )\,\mathrm{GRup}(\ ,\ )+\mathrm{GRupc}(\ ,\ )\,\mathrm{GRup}(\ ,\ )\,\mathrm{GRupc}(\ ,\ )-\mathrm{GRupc}(\ ,\ )\,\mathrm{GRupc}(\ ,\ )\,\mathrm{GRup}(\ ,\ )$$
$$-\mathrm{GRupc}(\ ,\ )\,\mathrm{GRupc}(\ ,\ )\,\mathrm{GRup}(\ ,\ )-\mathrm{GRupc}(\ ,\ )\,\mathrm{GRupc}(\ ,\ )\,\mathrm{GRup}(\ ,\ )-\mathrm{GRupc}(\ ,\ )\,\mathrm{GRupc}(\ ,\ )\,\mathrm{GRupc}(\ ,\ )$$
$$-\mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(i\uparrow,j\uparrow)\,\mathrm{GRup}(k\downarrow,j\downarrow)+\mathrm{GRupc}(k\downarrow,i\uparrow)\,\mathrm{GRup}(i\uparrow,j\downarrow)\,\mathrm{GRupc}(k\uparrow,j\uparrow)-\mathrm{GRupc}(i\uparrow,i\uparrow)\,\mathrm{GRupc}(k\uparrow,j\uparrow)\,\mathrm{GRup}(k\downarrow,j\downarrow)$$
$$-\mathrm{GRupc}(k\downarrow,j\uparrow)\,\mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(i\uparrow,j\downarrow)-\mathrm{GRupc}(k\uparrow,j\downarrow)\,\mathrm{GRupc}(k\downarrow,i\uparrow)\,\mathrm{GRup}(i\uparrow,j\uparrow)-\mathrm{GRupc}(k\uparrow,j\downarrow)\,\mathrm{GRupc}(k\downarrow,j\uparrow)\,\mathrm{GRupc}(i\uparrow,i\uparrow)$$

$$+\mathrm{GRupc}(l\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\uparrow)\,\mathrm{GRup}(l\downarrow,j\downarrow)-\mathrm{GRupc}(l\uparrow,i\downarrow)\,\mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\uparrow)\,\mathrm{GRup}(l\downarrow,j\downarrow)$$
$$-\mathrm{GRupc}(l\uparrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\uparrow)\,\mathrm{GRupc}(l\downarrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\downarrow)-\mathrm{GRupc}(l\downarrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\downarrow)\,\mathrm{GRupc}(l\uparrow,j\uparrow)$$
$$+\mathrm{GRupc}(l\downarrow,i\downarrow)\,\mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\downarrow)\,\mathrm{GRupc}(l\uparrow,j\uparrow)-\mathrm{GRupc}(l\downarrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\downarrow)\,\mathrm{GRupc}(l\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\uparrow)$$

$$+\mathrm{GRupc}(l\uparrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\uparrow)\,\mathrm{GRup}(l\downarrow,j\downarrow)\,\mathrm{GRupc}(k\downarrow,i\uparrow)-\mathrm{GRupc}(l\downarrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\downarrow)\,\mathrm{GRup}(l\uparrow,j\uparrow)\,\mathrm{GRupc}(k\downarrow,i\uparrow)$$
$$+\mathrm{GRupc}(l\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\uparrow)\,\mathrm{GRup}(l\downarrow,j\downarrow)\,\mathrm{GRupc}(k\uparrow,i\downarrow)-\mathrm{GRupc}(l\downarrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\downarrow)\,\mathrm{GRup}(l\uparrow,j\uparrow)\,\mathrm{GRupc}(k\uparrow,i\downarrow)$$
$$+\mathrm{GRupc}(l\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\downarrow)\,\mathrm{GRupc}(l\downarrow,j\uparrow)-\mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\downarrow)\,\mathrm{GRup}(l\uparrow,i\downarrow)\,\mathrm{GRupc}(l\downarrow,j\uparrow)$$
$$+\mathrm{GRupc}(l\downarrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\uparrow)\,\mathrm{GRupc}(l\uparrow,j\downarrow)-\mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\uparrow)\,\mathrm{GRup}(l\downarrow,i\downarrow)\,\mathrm{GRupc}(l\uparrow,j\downarrow)$$

$$+\mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,i\downarrow)\,\mathrm{GRupc}(l\uparrow,j\uparrow)\,\mathrm{GRup}(l\downarrow,j\downarrow)+\mathrm{GRupc}(l\downarrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\uparrow)\,\mathrm{GRupc}(l\uparrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\downarrow)$$
$$+\mathrm{GRupc}(l\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\downarrow)\,\mathrm{GRupc}(l\downarrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\uparrow)$$

$$+\mathrm{GRupc}(k\downarrow,i\uparrow)\,\mathrm{GRupc}(k\uparrow,i\downarrow)\,\mathrm{GRupc}(l\downarrow,j\uparrow)\,\mathrm{GRupc}(l\uparrow,j\downarrow)$$

$$+\mathrm{GRupc}(k\downarrow,i\uparrow)\,\mathrm{GRupc}(k\uparrow,i\downarrow)\,\mathrm{GRupc}(l\uparrow,j\uparrow)\,\mathrm{GRup}(l\downarrow,j\downarrow)+\mathrm{GRupc}(k\downarrow,i\uparrow)\,\mathrm{GRupc}(l\uparrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\downarrow)\,\mathrm{GRupc}(l\downarrow,j\uparrow)$$
$$+\mathrm{GRupc}(k\downarrow,i\uparrow)\,\mathrm{GRupc}(l\downarrow,i\downarrow)\,\mathrm{GRup}(k\uparrow,j\uparrow)\,\mathrm{GRupc}(l\uparrow,j\downarrow)+\mathrm{GRupc}(k\uparrow,i\downarrow)\,\mathrm{GRupc}(l\downarrow,j\uparrow)\,\mathrm{GRupc}(l\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\downarrow)$$
$$+\mathrm{GRupc}(k\uparrow,i\downarrow)\,\mathrm{GRupc}(l\uparrow,j\downarrow)\,\mathrm{GRupc}(l\downarrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,j\uparrow)+\mathrm{GRupc}(l\downarrow,j\uparrow)\,\mathrm{GRupc}(l\uparrow,j\downarrow)\,\mathrm{GRupc}(k\uparrow,i\uparrow)\,\mathrm{GRup}(k\downarrow,i\downarrow)$$

$$G_f=\frac{1}{L^2}\sum_i\langle c_i^\dagger c_{i+\vec{R}_m}+c^\dagger_{\underbrace{i+\vec{R}_m}_{(\frac{L-1}{2},\frac{L-1}{2})}}c_i\rangle$$

不知道咋算，先分别测测吧.

$$G_{f\text{-}uu}=\frac{1}{L^2}\sum_i\langle c_{i\uparrow}^\dagger c_{i+\vec{R}_m\uparrow}+c^\dagger_{i+\vec{R}_m\uparrow}c_{i\uparrow}\rangle=\frac{1}{L^2}\sum_i \left(\mathrm{GRupc}(i+\vec{R}_m\uparrow,i\uparrow)+\mathrm{GRupc}(i\uparrow,i+\vec{R}_m\uparrow)\right)$$
$$G_{f\text{-}ud}=\frac{1}{L^2}\sum_i \left(\mathrm{GRupc}(i+\vec{R}_m\downarrow,i\uparrow)+\mathrm{GRupc}(i\uparrow,i+\vec{R}_m\downarrow)\right)$$
$$G_{f\text{-}du}=\frac{1}{L^2}\sum_i \left(\mathrm{GRupc}(i+\vec{R}_m\uparrow,i\downarrow)+\mathrm{GRupc}(i\downarrow,i+\vec{R}_m\uparrow)\right)$$
$$G_{f\text{-}dd}=\frac{1}{L^2}\sum_i \left(\mathrm{GRupc}(i+\vec{R}_m\downarrow,i\downarrow)+\mathrm{GRupc}(i\downarrow,i+\vec{R}_m\downarrow)\right)$$

可观测量　　结构因子

相当于格点平方 N^2　格点-格点

$$S_{sc}(L)=\frac{1}{L^4}\sum_{ij}\langle\Delta_i^\dagger\Delta_j\rangle=\frac{1}{L^4}\sum_{ij}\langle c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{j\downarrow}c_{j\uparrow}\rangle,\quad \Delta_i=c_{i\downarrow}c_{i\uparrow}$$

$$M_2\equiv\frac{1}{N^2}\sum_{ij}\langle\Delta_i^\dagger\Delta_j\rangle=S_{sc},\quad M_4\equiv\frac{1}{N^4}\sum_{ijkl}\langle\Delta_i^\dagger\Delta_j^\dagger\Delta_k\Delta_l\rangle$$

$$G_f(L)=\frac{1}{L^2}\sum_i\langle c_i^\dagger c_{\underbrace{i+\vec{R}_m}_{(\frac{L-1}{2},\frac{L-1}{2})}}+c^\dagger_{i+\vec{R}_m}c_i\rangle\propto L^{-(2+\eta_f)}$$

fermion反常维度 $\frac{1}{3}$

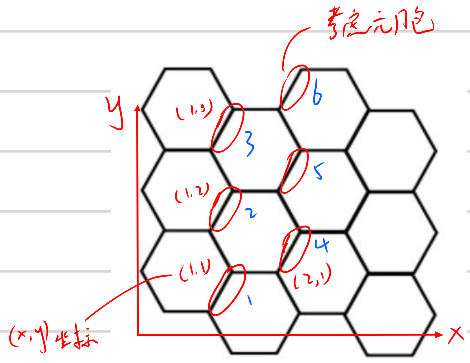
PREQ

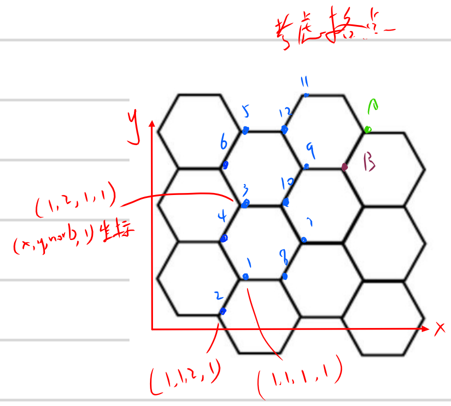
用 orderparameter (gri, file) 直接输出 M_2, M_4, Gf-uu, Gf-ud, Gf-du, Gf-dd chiral Heisenberg

$$H = -\sum_{\langle ij\rangle,\sigma}\left(t c_{i\sigma}^{\dagger} c_{j\sigma} + t^{*} c_{j\sigma}^{\dagger} c_{i\sigma}\right) + U\sum_{i}\left(n_{i\uparrow}-\frac{1}{2}\right)\left(n_{i\downarrow}-\frac{1}{2}\right)$$

$U>0$　　图像横轴变量为 U/t

六角晶格





可观测量　$S(\vec{q}) \equiv \frac{1}{N^2}\sum_{ij} e^{i\vec{q}(\vec{r}_i-\vec{r}_j)}\langle m_i^{(z)} m_j^{(z)}\rangle$

格点　　元胞

$$m_i^{(z)} \equiv S^z_{i,A} - S^z_{i,B}$$

一个元胞上两格点的自旋差

$$= \left(S_{i\uparrow,A} - S_{i\downarrow,A}\right) - \left(S_{i\uparrow,B} - S_{i\downarrow,B}\right)$$

$$= \vec{c}_{i,A}^{\,\dagger}\sigma^z \vec{c}_{i,A} - \vec{c}_{i,B}^{\,\dagger}\sigma^z \vec{c}_{i,B}$$