Optimal Control of Trapped Ions

The Noise-Resilient Entangling Gates

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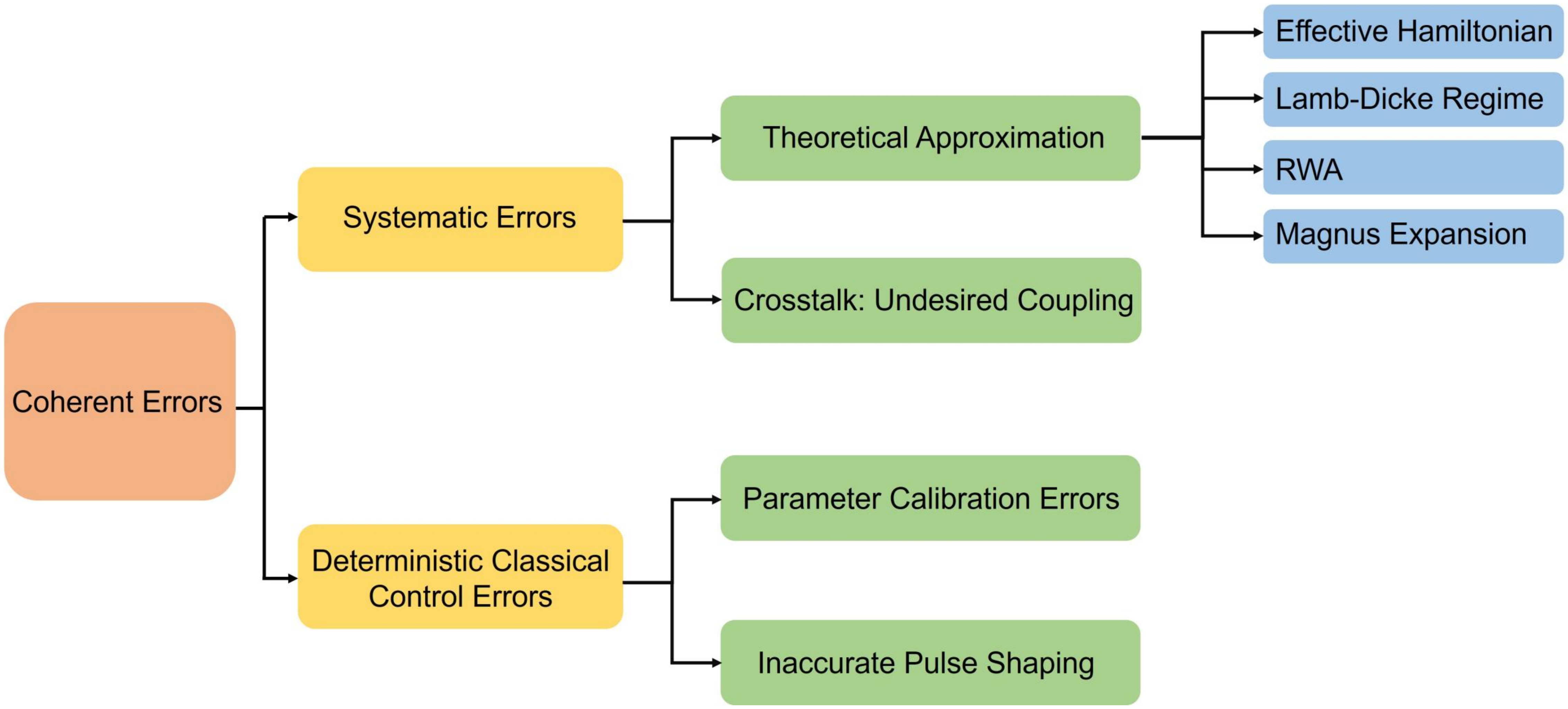
Contents

1. The Errors Causing Infidelity

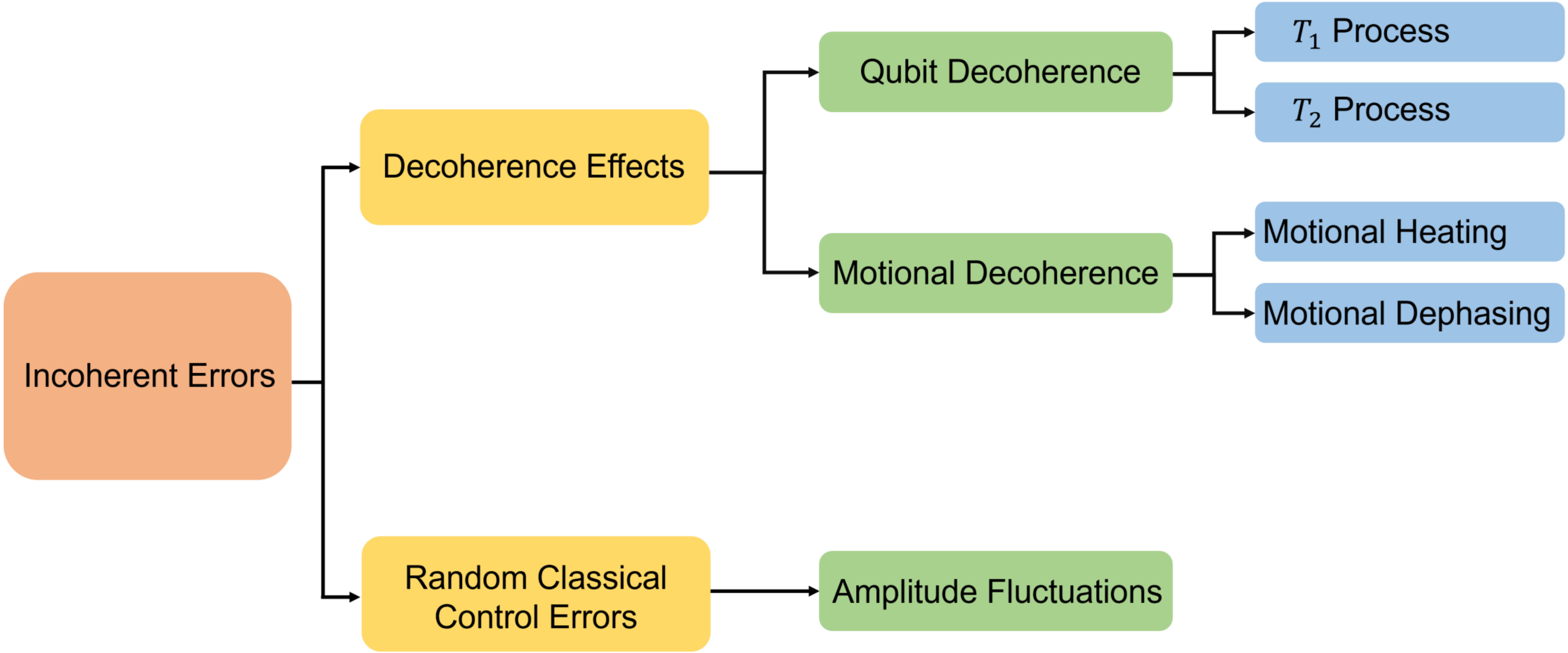
Fidelity: $F \equiv \langle \psi_{\mathrm{ideal}} | \rho_{\mathrm{actual}} | \psi_{\mathrm{ideal}} \rangle$

Infidelity: $I = 1 - F$ ⟸ Errors $\begin{cases} \text{Coherent Errors} \\ \text{Incoherent Errors} \end{cases}$

1.1 Coherent Errors



1.2 Incoherent Errors



2.1 The Conventional Mølmer-Sørensen Gate[1]

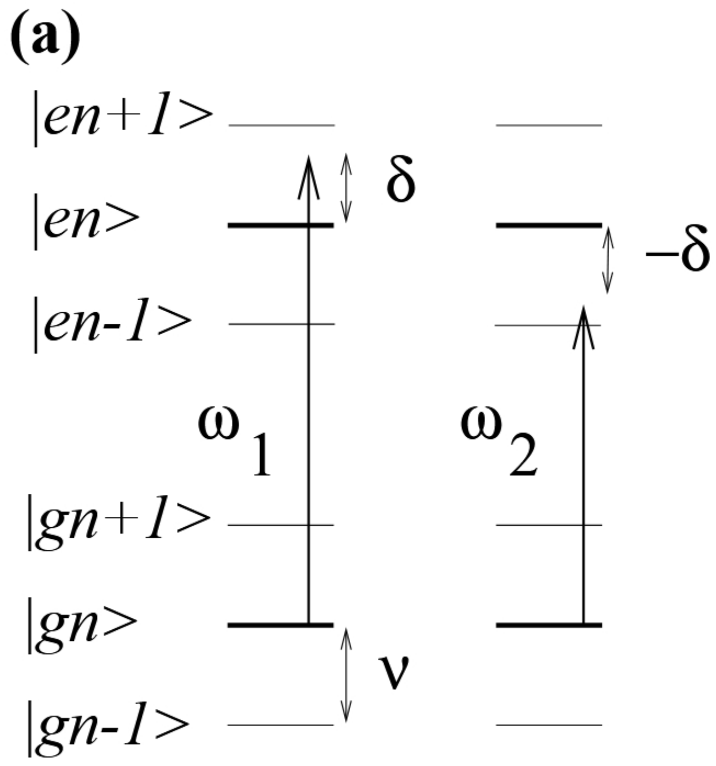
- **Trapped-Ion Hamiltonian**

$$H_0 = \hbar\nu\left(a^\dagger a + 1/2\right) + \hbar\omega_{eg}\sum_i \sigma_{zi}/2$$

$$H_{\text{int}} = \sum_i \frac{\hbar\Omega_i}{2}\left(\sigma_{+i}e^{\mathrm{i}\left(\eta_i\left(a+a^\dagger\right)-\omega_i t\right)} + h.c.\right)$$

- **Cirac-Zoller gate** ~ resonant
  - blue: $\omega = \omega_{eg} + \nu$
  - red: $\omega = \omega_{eg} - \nu$

- **Mølmer-Sørensen (MS) gate** ~ off-resonant
  - $\omega_1 = \omega_{eg} + \delta$
  - $\omega_2 = \omega_{eg} - \delta$



[1] 10.1103/PhysRevLett.82.1971

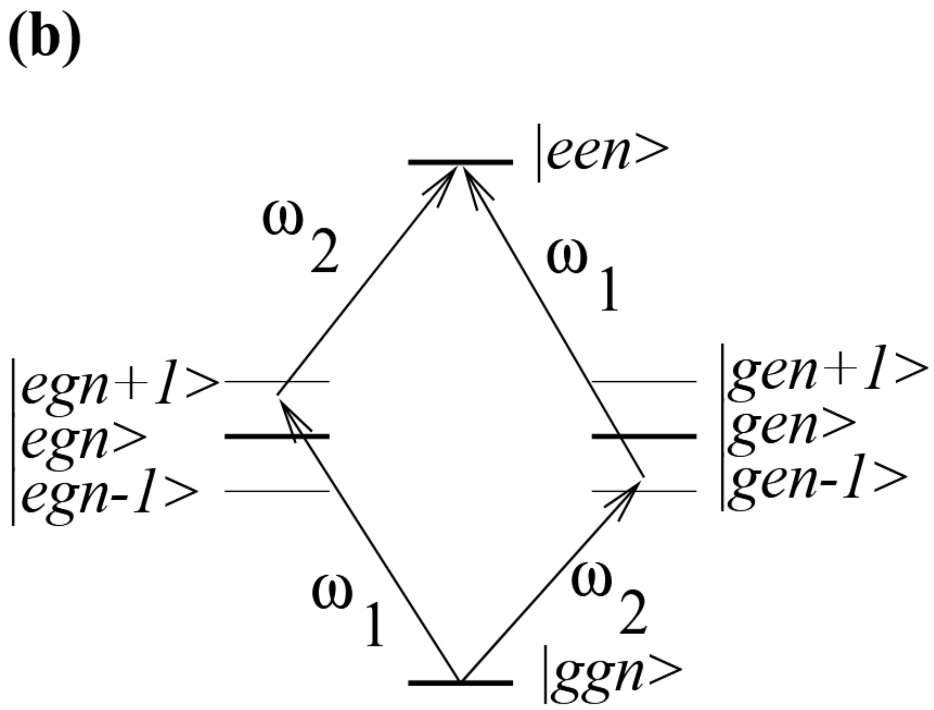
2.1 The Conventional Mølmer-Sørensen Gate[1]

• **Second order perturbation theory**

$$\left(\frac{\tilde{\Omega}}{2}\right)^2 = \frac{1}{\hbar^2}\left|\sum_m \frac{\langle een|H_{\text{int}}|m\rangle\langle m|H_{\text{int}}|ggn\rangle}{E_{ggn}+\hbar\omega_i - E_m}\right|^2$$

$$m = |egn+1\rangle, |gen-1\rangle$$

$$\tilde{\Omega} = -\frac{(\Omega\eta)^2}{2(\nu-\delta)}$$



[1] 10.1103/PhysRevLett.82.1971

2.2 The Polychromatic Mølmer-Sørensen Gate[2]

- **MS Hamiltonian**

$$H(t) = \left(\Upsilon(t)a + \Upsilon^*(t)a^\dagger\right)S_x, \qquad S_x = \sum_j \sigma_x^{(j)}, \Upsilon(t) = \eta\Omega\exp(\mathrm{i}\delta t)$$

- **Propagator**

$$U_K = \exp(-\mathrm{i}((f(t)a + f^*(t)a^\dagger)S_x - g(t)S_x^2))$$

$$f(t) = \int_0^t dt'\Upsilon(\mathrm{t}'), g(t) = \mathrm{Im}[\int_0^t dt'\Upsilon(\mathrm{t}')f^*(t')]$$

- **Lindblad master equation**

$$\mathcal{L}[\circ] = -\mathrm{i}[H(t),\circ] + \sum_{j=+,-,d} \gamma_j\left(E_j \circ E_j^\dagger - \frac{1}{2}\{E_j^\dagger E_j,\circ\}\right)$$

$$E_+ = a^\dagger$$
$$E_- = a$$
$$E_d = a^\dagger a$$

Interaction picture →

$$\tilde{E}_+ = a^\dagger + if(t)S_x$$
$$\tilde{E}_- = a - if^*(t)S_x$$
$$\tilde{E}_d = a^\dagger a + i\left(f(t)a - f^*(t)a^\dagger\right)S_x + |f(t)|^2 S_x^2$$

[2] 10.1088/1367-2630/18/12/123007

2.2 The Polychromatic Mølmer-Sørensen Gate[2]

- **Optimization**

constrain $\langle f \rangle = 0$
minimize $\langle |f|^2 \rangle$

$\Upsilon_{\mathrm{p}} = \sum_{j=1}^{m} c_j \omega \exp(\mathrm{i} j \omega t)$

maximally entangling unitary

$$\min_{\{c_j\}} \left(\sum_{j=1}^{m} \frac{|c_j|^2}{j^2} \mid \sum_{j=1}^{m} \frac{c_j}{j} = 0, \sum_{j=1}^{m} \frac{|c_j|^2}{j} = \frac{1}{16} \right)$$

optimal coefficient c_j^{opt}

- **Polychromatic MS Hamiltonian**

$$H_{\mathrm{poly}} = \hbar \delta S_x \sum_{j=1}^{N} c_j (a^\dagger e^{ij\delta t} + a e^{-ij\delta t})$$

protect against

- motional heating
- motional dephasing

[2] 10.1088/1367-2630/18/12/123007

2.3 Generally Noise-resilient Entangling Gates[3]

- **Many-ion system with multi-mode structure**

$$H_0 = \sum_{j=1}^{N} \frac{\omega_j}{2} \sigma_z^{(j)} + \sum_{l=1}^{M} \nu_l a_l^\dagger a_l$$

$$H(t) = \sum_{j=1}^{N} \Upsilon_j(t) \sigma_+^{(j)} \prod_{l=1}^{M} e^{i\eta_{jl}\left(a_l + a_l^\dagger\right)} + h.c.$$

- the entanglement is achieved via the exchange of virtual phonons
- interaction picture
- RWA

$$H(t) = \sum_{j=1}^{N} \sigma_y^{(j)} \sum_{l,k>0} \frac{F_{l,k}^{(j)}(t)}{\eta_{jl}} D_{l,k} \prod_{l' \neq l} D_{l',0} + h.c.$$

- Operators $D_{l,k}$ capture the k-th order sideband transitions of the motional mode l.

- Driving patterns $F_{l,k}^{(j)}(t)$ account for finite detuning and temporal modulation that can be used to achieve the desired robustness

[3] arXiv: 2404.12961

2.3 Generally Noise-resilient Entangling Gates[3]

beyond the Lamb-Dicke regime →

- higher-order phonon processes
- the phonon exchange processes in the non-addressed vibrational modes

→

- 1st-order sideband ($k = 1$)
  2nd-order sidebands ($k = 2$)
- simultaneously drive all modes

$$H_c(t) = \sum_{j=1}^{N} \sigma_y^{(j)} \left(\frac{F_{1,1}^{(j)}(t)}{\eta_{j1}} D_{1,1} D_1 + \sum_{l=1}^{M} \frac{F_{l,2}^{(j)}(t)}{\eta_{jl}} D_{l,2} D_l \right) + h.c.$$

$$F_{1,1}^{(1)} = F_{1,1}^{(2)} = \Omega \left(e^{2\mathrm{i}\delta t} - \frac{3}{2} e^{3\mathrm{i}\delta t} \right)$$ ~ bichromatic modulation ~ against motional heating

$$F_{l,2}^{(1)} = \mathrm{sign}(\eta_{1l}) \Omega \frac{\tilde{\eta}_l}{\eta_{1l}} e^{\mathrm{i}\delta t}$$

$$F_{l,2}^{(2)} = \mathrm{sign}(\eta_{2l}) \Omega \frac{\tilde{\eta}_l}{\eta_{2l}} e^{\mathrm{i}\delta t}$$

[3] arXiv: 2404.12961

2.4 Amplitude-noise-resilient Entangling Gates[4]

• The sensitivity to amplitude in harmonic trap

For the MS Hamiltonian of a pair of trapped ions and an ideal harmonic bus mode:

• the gate dynamics corresponds to a phase-space trajectory, which is a closed loop with length proportional to Ω_R

• the Rabi-angle Φ_R of the effective S_x^2-interaction in this dynamics is proportional to the area enclosed by the loop

$$\Phi_R = \Omega_R^2 \, \mathrm{Im}\left[\int_0^T d\tau \widetilde{\Upsilon}(\tau) \int_0^\tau d\tau' \widetilde{\Upsilon}^*(\tau')\right]$$

no choice of modulation $\Upsilon(t)$ can alter this quadratic dependence on Ω_R →

$$\frac{d\Phi_R}{\Phi_R} \sim \frac{d\Omega_R}{\Omega_R}$$

[4] arXiv: 2407.03047

2.4 Amplitude-noise-resilient Entangling Gates[4]

- **The sensitivity to amplitude in weak anharmonic trap**
  - trap geometry: DC control electrodes are placed directly underneath the ions

quartic potential: $\frac{1}{2}m\omega^2(z^2 + z^4/\xi^2)$

  - design $\Upsilon(t)$ to make the correction terms induced by the quartic potential to compensate for phase variations induced by fluctuations in Ω_R

$$\frac{d\Phi_R}{\Phi_R} \sim \left(\frac{d\Omega_R}{\Omega_R}\right)^2$$

[4] arXiv: 2407.03047

3. Summary

- The conventional MS gate: independent of the initial motional quantum number

- The polychromatic MS gate: against motional heating and motional dephasing

- Generally noise-resilient entangling gate scheme: against motional heating in hot trapped ion chain with complex multi-mode structure

- Amplitude-noise-resilient gate scheme: against the random amplitude fluctuation

Thank you!