The 12th Workshop on Quantum Many-Body Computation

Finite-time Scaling beyond the Kibble-Zurek Prerequisite: Driven Critical Dynamics in Strongly Interacting Dirac Systems[1]

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Introduction

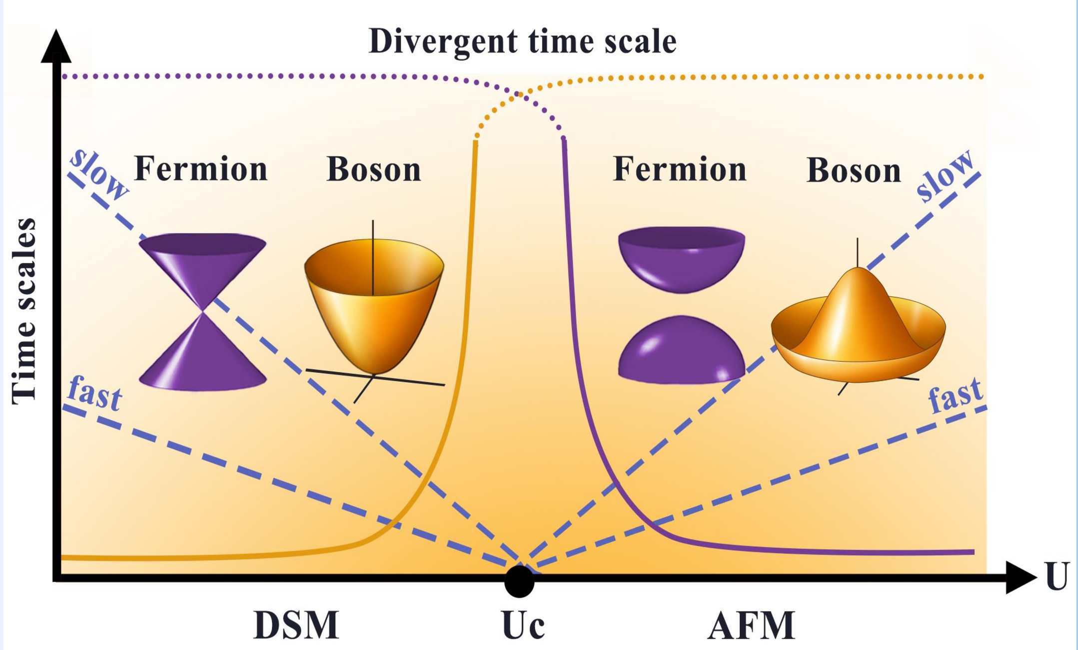
• **Driven dynamics:** Linearly driving the system across the critical point
--Scaling theory in conventional bosonic quantum critical point (QCP):
(1) Kibble-Zurek mechanism (KZM): generation and scaling of topological defects after linear annealing[2,3]
(2) Finite-time scaling (FTS): scaling form in the whole driven process[4,5]
(3) Real- and imaginary-time driven dynamics share the same scaling form[6-8]

• **Dirac fermionic QCP:** Triggered by the interplay between fluctuations of gapless Dirac fermions and order-parameter bosons[9,10]
--Graphene, Weyl/Dirac semimetal, surface of topological insulator
--Gross-Neveu universality class[11-13]

• **This work:** For the first time we investigate the driven critical dynamics of two representative Dirac quantum critical points, belonging to chiral Heisenberg and chiral Ising universality class respectively, via the determinant quantum Monte Carlo method[14].

Imaginary-time driven dynamics

• Sketch of the phase diagram



• The protocol for driven dynamics with different initial states:

$$U = U_0 \pm R\tau$$

U: interaction strength.
τ: imaginary time.
R: driving rate.
$+(-)$: disordered (ordered) initial state at U_0.

We linearly vary the interaction strength along the imaginary-time direction to cross the QCP from both the Dirac Semimetal (DSM) and Mott insulator phases.

• Imaginary-time Schrödinger equation[6-8]:

$$-\frac{\partial}{\partial\tau}|\psi(\tau)\rangle = H(\tau)|\psi(\tau)\rangle$$

Formal solution

$$|\psi(\tau)\rangle = U(\tau,0)|\psi(0)\rangle$$

Time evolution operator

$$U(\tau,0) = \mathrm{T}\exp[-\int_0^\tau d\tau' H(\tau')]$$

T: time-ordering operator

Summary

1. We confirm that the driven dynamics of Gross-Neveu universality class is described by the FTS form.

2. We not only successfully generalize the KZM and FTS to critical systems with joint fluctuations of gapless fermions and bosons, but also extend the application conditions of KZM.

3. We demonstrate that the nonequilibrium scaling form is capable of determining the critical exponents in Dirac QCP, providing an effective method to deciphering quantum critical properties in terms of driven dynamics.

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Chiral Heisenberg criticality

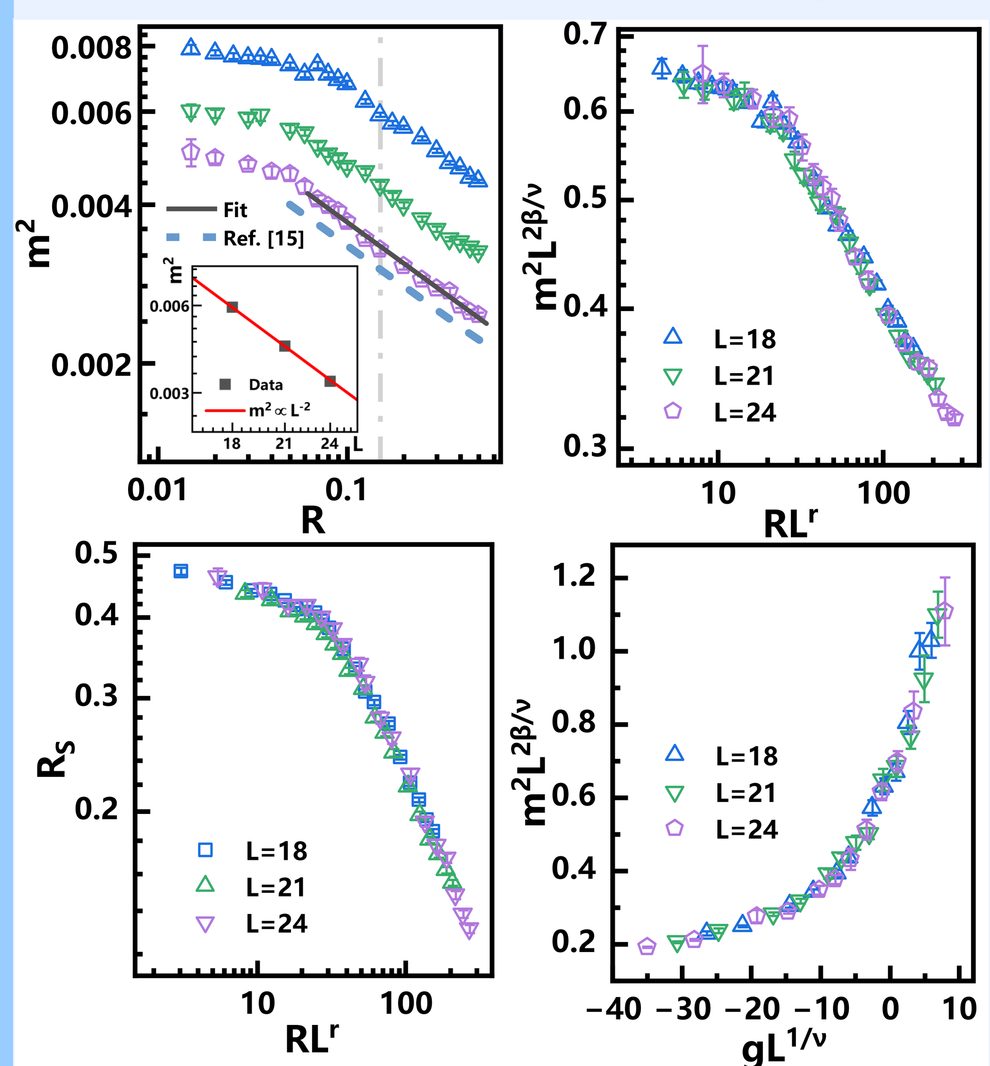
Spin-1/2 Hubbard model on the half-filled 2D honeycomb lattice with Hamiltonian[15]:

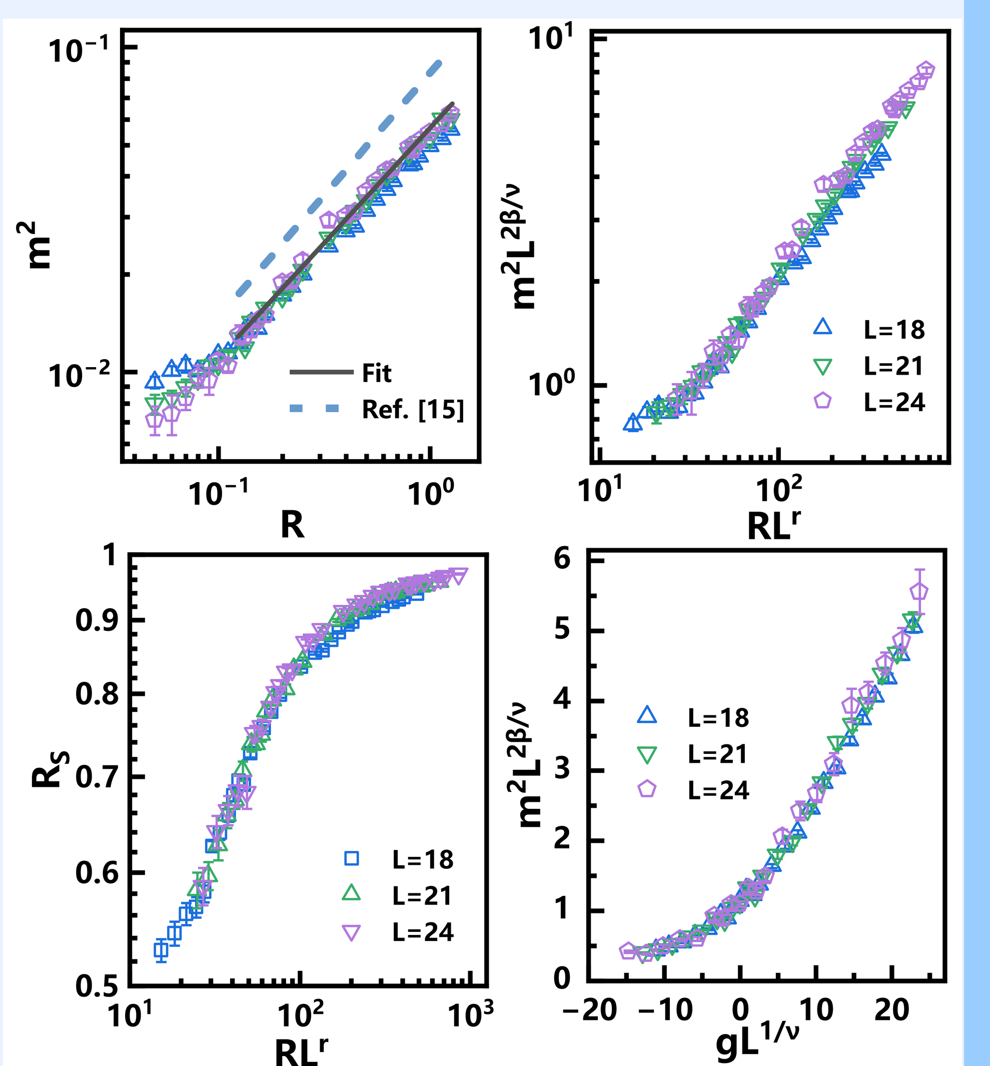
$$H = -t\sum_{\langle ij\rangle,\sigma} c_{i\sigma}^\dagger c_{j\sigma} + U\sum_i (n_{i\uparrow}-\frac{1}{2})(n_{i\downarrow}-\frac{1}{2})$$

Structure factor: $S(\boldsymbol{q}) \equiv \frac{1}{L^{2d}}\sum_{i,j} e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_i-\boldsymbol{r}_j)}\langle S_i^{(z)} S_j^{(z)}\rangle$, $S_i^{(z)} \equiv \vec{c}_{i,A}^{\,\dagger}\sigma^z\vec{c}_{i,A} - \vec{c}_{i,B}^{\,\dagger}\sigma^z\vec{c}_{i,B}$

Order parameter: $m^2 \equiv S(\boldsymbol{0})/4$

Correlation ratio[16]: $R_s \equiv 1 - S(\Delta\boldsymbol{q})/S(\boldsymbol{0})$





From DSM initial state

For large R and fixed L

$$m^2 \propto R^{(2\beta-d\nu)/\nu r}$$

For fixed large R

$$m^2 \propto L^{-d}$$

The scaling form must satisfy[17]

$$m^2(R,L,g) \propto L^{-d}R^{(2\beta-d\nu)/\nu r}\mathcal{F}(RL^r, gL^{1/\nu})$$

Dimensionless variable R_S obeys

$$R_S(R,L,g) \propto f_1(RL^r, gL^{1/\nu})$$

From AFM initial state

For large R and fixed L

$$m^2 \propto R^{2\beta/\nu r}$$

For small R

$$m^2 \propto L^{-2\beta/\nu}$$

The scaling form must satisfy

$$m^2(R,L,g) \propto R^{2\beta/\nu r}\mathcal{M}(RL^r, gL^{1/\nu})$$

Dimensionless variable R_S obeys

$$R_S(R,L,g) \propto f_2(RL^r, gL^{1/\nu})$$

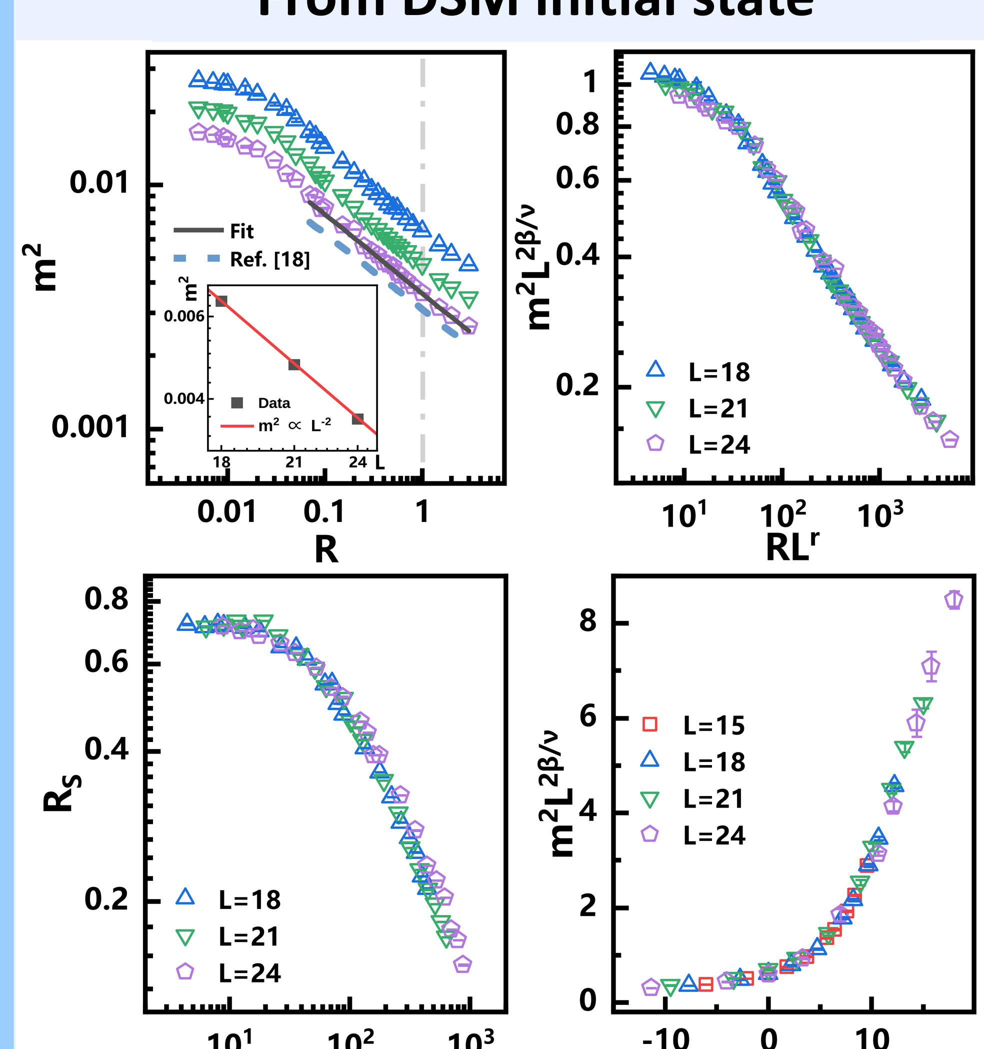
Chiral Ising criticality

Spinless fermion model on the half-filled 2D honeycomb lattice with Hamiltonian[18]:

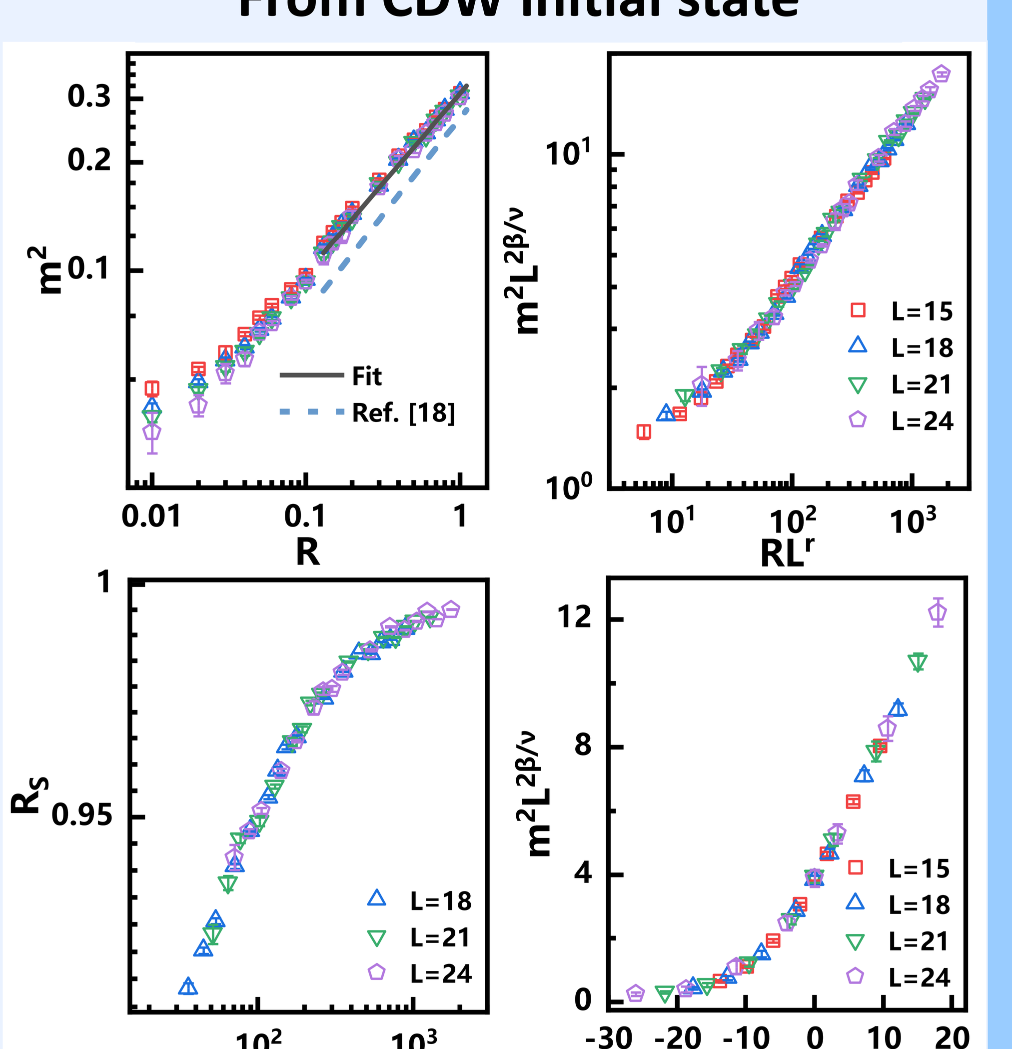
$$H = -t\sum_{\langle ij\rangle} c_i^\dagger c_j + V\sum_{\langle ij\rangle}(n_i - \frac{1}{2})(n_j - \frac{1}{2})$$

Structure factor: $S(\boldsymbol{q}) \equiv \frac{1}{L^{2d}}\sum_{i,j} e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_i-\boldsymbol{r}_j)}\langle n_i n_j\rangle$, $n_{i,A} \equiv c_{i,A}^\dagger c_{j,A}$

From DSM initial state



From CDW initial state



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